How to obtain large gradients in composite media and the Neumann-Poincaré operator

Eric Bonnetier
Laboratoire Jean Kuntzmann
Université de Grenoble-Alpes
Eric.Bonnetier@imag.fr

Abstract.
In this course, I will present work that has been developed over the last decade concerning large gradients in composite media made of inclusions with smooth boundaries embedded in a matrix phase. The problems originates from a question of Ivo Babuška, who was concerned with the possible onset of cracks in the narrow channels between close-to-touching inclusions in such composites. If large values of the gradients can be detrimental in mechanics, they may be beneficial in other contexts, such as plasmonics. There, large gradients may be the signature of localized resonant modes, which can be used to identify single molecules or to destroy cancerous cells. This resonant mechanism is also key in a form of cloaking known as claoking via anomalous localized resonance, hence the connection with inverse problems.

We will first discuss the case of the conduction equation in a medium made of 2 discs of conductivity k embedded in a matrix phase of conductivity 1, and separated by a distance δ. We will see how the parameters δ and k govern the pointwise behavior of the gradient of the voltage potential. I will survey the results concerning pointwise bounds on the gradient, that have been obtained by several groups, when $0 < k < \infty$, but also in the degenerate case when $k = \infty$.

Next, we will study the Neumann-Poincaré operator K^* associated to a system of 2 inclusions. This integral operator naturally appears when one seeks a representation of the voltage potential in terms of layer potentials. After recalling the basic theory concerning layer potentials, I will derive the system of integral equations

$$(\lambda I - K^*) \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix},$$

associated to a system of 2 close-to-touching discs. I will show how the spectral properties of this operator are related to the possible blow up of the gradient of the voltage potential.

Finally, I will describe how cloacking via anomalous localized resonance is related to the Neumann-Poincaré operator and to creating large gradients in composite media.